

# Transmission Pricing Software for Power Engineering Education

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**ABSTRACT:** This article presents a computer program that is used in a power system economics course in order to clarify the differences and the impact of eight transmission pricing and three tracing methodologies on transmission cost allocation. The software can graphically represent the allocation of transmission cost to network users. Thanks to its graphical user interface, the software is very friendly for the students. Moreover, this article presents an educational example that helps students understand all the calculations that are involved in transmission pricing. The software and the example-driven presentation have been proven very efficient in the education of students at the National Technical University of Athens (NTUA), Greece. In conclusion, the work presented in this article advances the use of computer application in the education of transmission pricing. © 2011 Wiley Periodicals, Inc. *Comput Appl Eng Educ* 22:410–428, 2014; View this article online at [wileyonlinelibrary.com/journal/cae](http://wileyonlinelibrary.com/journal/cae); DOI 10.1002/cae.20565

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## INTRODUCTION

One of the sixth semester required courses, of the 5-year undergraduate curriculum of Electrical and Computer Engineering at the National Technical University of Athens (NTUA), Greece, is the course of power system economics that includes the teaching of transmission pricing principles and methodologies. Transmission pricing is one of the major issues in transmission open access faced by the electric power industry in the context of deregulated electricity markets. *Transmission pricing* methodologies determine transmission cost allocation to transmission network users. The *transmission cost* includes all the transmission system costs: existing transmission system, operation, maintenance, and transmission expansion cost. The competitive environment of electricity markets necessitates wide access to transmission networks. Moreover, as power flows influence transmission charges, transmission pricing not only determines the right of entry but also encourages efficiencies in power markets. For example, transmission constraints could prevent an efficient and economic generating unit from being utilized. A proper transmission pricing scheme that considers

transmission constraints or congestion could motivate investors to build new transmission and/or generating capacity for improving the efficiency. Consequently, transmission pricing should provide signals toward the efficient use, operation, and expansion of the transmission network [1].

Different usage-based methods have been proposed for transmission cost allocation. Regardless of the market structure, it is important to accurately determine transmission usage in order to implement usage-based cost allocation methods. However, determining an accurate transmission usage could be difficult due to the nonlinear nature of power flow. This fact necessitates using approximate models, sensitivity indices, or tracing algorithms to determine the contributions to the network flows from individual users or transactions [1].

The problem of transmission cost allocation to network users in pool markets is divided into three sub-problems: (1) a load flow solution that may be representative of a certain load and generation pattern, (2) the allocation of transmission line power flows to each network user, and (3) the cost allocation to the already allocated flows. The allocation is solved assuming a percentage share of the transmission cost for generators and loads [2].

This article presents a novel approach to education in the field of transmission pricing. A computer program, called transmission pricing software (TPS), has been developed to present the effects of eight transmission pricing and three tracing

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methodologies on transmission cost allocation. This computer program is used for teaching transmission pricing in the context of power system economics course at the National Technical University of Athens (NTUA). More specifically, the eight transmission pricing methodologies are: MW-mile original, unused absolute MW-mile, unused zero counter flow MW-mile, unused reverse MW-mile, used absolute MW-mile, used zero counter flow MW-mile, used reverse MW-mile, and postage stamp method. The three tracing methods are: distribution factors, Bialek, and minimum power distance method.

The TPS has been implemented in MATLAB, because it integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation [3–10]. When designing the TPS, special care was given to its graphical user interface (GUI), as it helps the user arrive at the final solution by visualizing each step of the design process [4]. Moreover, GUIs are being increasingly used to provide users of computer simulations with a friendly and visual approach [5]. Indeed, its GUI makes TPS very friendly to the students. Using TPS, the students can see not only the final solution, that is, the cost allocation among the network users, but also important intermediate results, for example, the contribution of network users to transmission line flows.

The use of TPS is presented for two different power systems: Garver's 6-bus and IEEE RTS 24-bus system. These test examples help the students understand the impact on transmission cost allocation of various parameters, for example, the location of the user, the tracing method used, the pricing or not of the counter flows, and the generation bid. The visualization of the parameters and the results of the TPS enable an easier and deeper insight into the impact of transmission system parameters and pricing methodologies on transmission cost allocation.

In the following sections, the optimal power flow (OPF), the transmission pricing methods, and the tracing methods are first described, as they are the basis for the transmission pricing software, which is described after an educational example, followed by application examples on the two power systems.

## OPTIMAL POWER FLOW

Optimal power flow involves the optimization of an objective function subject to a set of physical and operating constraints. In this article, the OPF formulation of Ref. [11] is used. More specifically, the objective function is the minimization of the sum of the total production cost of the scheduled generating units plus the cost of the load not supplied. The equality constraints are (a) the dc load flow equation and (b) the generation and load balance. The inequality constraints are (a) the active power flow limits, (b) the generation limits, (c) the load not served limits, and (d) the voltage angle limits.

The solution of the OPF problem gives the power flows in all the transmission lines. Moreover, locational marginal prices (LMPs) are obtained within the OPF framework, as they are the dual variables associated with the active power balance equations [12]. When there is no transmission congestion, LMPs are the same at all buses, while when transmission congestion takes place (some lines reach their thermal limits), LMPs are differentiated and the congestion revenue is computed [11].

## TRANSMISSION PRICING METHODOLOGIES

Transmission pricing is an important issue in restructured power systems. The cost of the basic transmission services corresponds primarily to the fixed transmission cost that is also referred to as the existing system cost, or the embedded transmission facility cost. Different methods have been proposed for transmission pricing.

### Postage Stamp Method

The postage stamp method allocates the total transmission cost to network users (generators and loads) as follows [13]:

$$TC_t = TC \frac{P_t}{P_{\text{peak}}} \quad (1)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $TC$  is the total transmission cost,  $P_t$  is the power (production or consumption) of user  $t$  at the time of system peak load, and  $P_{\text{peak}}$  is the system peak load.

The postage stamp method does not require power flow calculations. Moreover, this method is independent of the transmission distance, supply, and delivery points or the loading on different transmission facilities caused by the network users.

### Original MW-Mile Method

The original MW-mile methodology may be regarded as the first pricing strategy proposed for the recovery of fixed transmission costs based on the actual use of transmission network as it is a dc load flow based method.

According to the original MW-mile method, the total transmission cost is allocated in each user  $t$  as follows [13]:

$$TC_t = TC \frac{\sum_{k \in K} c_k L_k MW_{t,k}}{\sum_{t \in T} \sum_{k \in K} c_k L_k MW_{t,k}} \quad (2)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $TC$  is the total transmission cost,  $c_k$  is the cost per MW per unit length of line  $k$ ,  $L_k$  is the length of line  $k$ ,  $MW_{t,k}$  is the power flow in line  $k$  due to user  $t$ ,  $T$  is the set of users, and  $K$  is the set of transmission lines.

There are variations of the original MW-mile method based on the charging of the unused transmission capacity and the pricing of the counter flows.

The difference in a transmission line capacity and the actual flow on that transmission line is called the unused transmission capacity [1]. Two options are available:

- (1) The *unused* transmission capacity methods. They charge the transmission users based on the whole transmission capacity, that is, the users pay not only for the actual line flows they cause but also for the unused transmission capacity, so these methods guarantee the full recovery of the fixed transmission cost. However, their drawback is that they do not motivate an efficient use of the transmission system.
- (2) The *used* transmission capacity methods. The transmission users are charged based on the actual line flows they cause, so these methods motivate an efficient use of the transmission system. However, their drawback is that the recovery of the fixed transmission costs is not guaranteed, because the actual line flows are usually smaller than the transmission line capacities [14].

Another important issue is the decision of pricing the counter flows. The directions of power flows caused by different users may be different on the same transmission line. Usually, the flows having the same direction with the net flow are called positive flows, or dominant flows. The flows having the different direction with the net flow are called negative flows or *counter flows* [15]. So, counter flows are associated with network users carrying power in opposite direction to the main flows. The counter flows are very helpful because they reduce the loading level of the facilities. Thus, the losses could be decreased and congestions could be avoided, so the available transfer capacity could be increased [14]. However, allowing negative allocation (credit) due to counter flows may not be easily accepted from network owner and some network users who will be charged a big portion of transmission fixed costs [15]. Three options are available:

- (1) The users who cause counter flows will pay for them, and the respective method is called *absolute* MW-mile.
- (2) The users who cause counter flows will get credit for them, and the respective method is called *reverse* MW-mile.
- (3) The users who cause counter flows will neither pay any charge nor get any credit for the counter flows, and the respective method is called *zero counter flow* MW-mile.

#### Unused Absolute MW-Mile Method

The unused absolute MW-mile method charges users based on the power flows they cause, irrespective of the power flow direction, that is, the users who cause counter flows will pay for them, so each user  $k$  has to pay [1]:

$$TC_t = \sum_{k \in K} C_k \frac{|F_{t,k}|}{\sum_{t \in T} |F_{t,k}|} \quad (3)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $T$  is the set of users, and  $K$  is the set of transmission lines.

#### Unused Reverse MW-Mile Method

In the unused reverse MW-mile, users get credit for the counter flows they cause. More specifically, the charge for user  $t$  is [14]

$$TC_t = \sum_{k \in K} C_k \frac{F_{t,k}}{\sum_{t \in T} F_{t,k}} \quad (4)$$

where  $TC_t$  is the cost allocated to user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $T$  is the set of users, and  $K$  is the set of transmission lines.

#### Unused Zero Counter Flow MW-Mile Method

The unused zero counter flow MW-mile method charges the users who use the network only in the same direction of the net power flow. So users responsible for the counter flows neither pay any charge nor get any credit for the counter flows. The payments are as follows [16]:

$$TC_t = \sum_{k \in K} C_k \frac{F_{t,k}}{\sum_{t \in T} F_{t,k}}, \quad \forall F_{t,k} > 0 \quad (5)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $T$  is the set of users, and  $K$  is the set of transmission lines.

#### Used Absolute MW-Mile Method

In the used absolute MW-mile method, the charge for user  $t$  becomes [1]

$$TC_t = \sum_{k \in K} C_k \frac{|F_{t,k}|}{F_{k,\max}} \quad (6)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $F_{k,\max}$  is the capacity of line  $k$ , and  $K$  is the set of transmission lines.

#### Used Reverse MW-Mile Method

In the used reverse MW-mile method, the charge for user  $t$  is [14]

$$TC_t = \sum_{k \in K} C_k \frac{F_{t,k}}{F_{k,\max}} \quad (7)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $F_{k,\max}$  is the capacity of line  $k$ , and  $K$  is the set of transmission lines.

#### Used Zero Counter Flow MW-Mile Method

In the used zero counter flow MW-mile method, the users have to pay only for the positive flows and nothing for the counter flows. The charge for user  $t$  is as follows [14]:

$$TC_t = \sum_{k \in K} C_k \frac{F_{t,k}}{F_{k,\max}}, \quad \forall F_{t,k} > 0 \quad (8)$$

where  $TC_t$  is the cost allocated to network user  $t$ ,  $C_k$  is the cost of line  $k$ ,  $F_{t,k}$  is the power flow on line  $k$  caused by user  $t$ ,  $F_{k,\max}$  is the capacity of line  $k$ , and  $K$  is the set of transmission lines.

### TRACING METHODS

Tracing methods determine the contribution of transmission users to transmission usage [1]. In an open access electricity market, the problem of tracing electricity gains importance as its solution could enhance transparency in the operation of the transmission system [1]. Tracing methods are used for transmission pricing and recovering fixed transmission costs. Three tracing methods are discussed here: (a) distribution factors, (b) Bialek, and (c) minimum power distance method. Another tracing method, often met in the literature, is Kirschen's tracing method [17].

#### Distribution Factors Method

Distribution factors based on dc power flows can be used as an efficient tool for measuring transmission usage. So, their use is limited to the active power flow [15]. The concept behind the use of distribution factors is to find out how a particular generator or load influences the power flow over particular transmission lines. There are three types of distribution factors [18]:

- (1) Generation shift distribution factors (GSDF). They measure the incremental use of the transmission network by generators or loads, that is, they provide line flow changes due to a change in generation or consumption. GSDF depend essentially on the network electrical parameters (reactances in particular) and the election of the reference bus, but they are independent of the operational conditions of the system. However, to determine the impacts on the network of the different injections, it is necessary to know the direction of the power flow in each branch in the study condition [18].
- (2) Generalized generation distribution factors (GGDF). They measure the total network use, not incremental one, produced by generator injections. They determine the impact of each generator on active power flows of transmission lines; thus GGDF can be negative as well. GGDF depend on line parameters and the studied operational conditions, but they are independent of the reference bus location.
- (3) Generalized load distribution factors (GLDF). They measure the total network use by negative generation injections, corresponding to loads. GLDF depend on line parameters and the studied operational conditions, but they are independent of the reference bus location.

### Bialek Method

Bialek method allocates the transmission usage (transmission line flows) to individual generators and/or loads by analyzing the topology of line flows. The method allows tracing where the output of every generator goes, or input to every load comes from, assuming that nodal inflows are shared proportionally between the outflows [19]. The resulting topological distribution factors allow determining the network usage by any generator and/or load by summing up the shares of each generator/load in every line flow. As the shares are always positive, no counter flow problems are encountered and all the charges to the network users are positive. The main advantage of the method is in its simplicity.

Bialek tracing algorithm has two versions [19]:

- (1) *Bialek upstream algorithm*. It allocates the transmission line flows to individual generators, that is, it assesses the contributions of individual generators to individual line flows.
- (2) *Bialek downstream algorithm*. It allocates the transmission line flows to individual loads, that is, it assesses the contributions of individual loads to individual line flows.

### Minimum Power Distance Method

The minimum power distance method assumes an economic principle according to which electricity flows through paths that minimize the total MW-km covered in the entire power system in order to obtain a linear programming model that is useful for large networks [20]. In this method, the term of *power distance* is introduced, which, if used within the linear programming model, provides an allocation of generation to loads that induces a flow decomposition such that each transmission line flow is broken in parcels of the (as much as possible) same orientation, that is, the counter flows are minimized [20]. This feature and the fact that such a model runs on moderate computing time for large networks makes it a practical tool for transmission cost allocation. Briefly, according to the minimum power

distance method, for each minimum power distance transaction from each generator to the corresponding load, the method allows to decompose the line flows in partial flows that each transaction causes in every line. Thus, transmission line usage by generators and loads is evaluated.

## EDUCATIONAL EXAMPLE

### Educational Objectives

This section presents an educational example that is currently used for the training of electrical engineering students in transmission pricing at NTUA. The educational objectives of this arithmetic example are the following:

- (1) The formulation of the DC OPF problem for a 6-bus test system.
- (2) The solution of the DC OPF problem using MATLAB Optimization Toolbox.
- (3) The determination of the contribution of transmission users to transmission usage based on distribution factors tracing method.
- (4) The computation of the cost allocation to transmission users based on MW-mile original pricing method.

### Problem

Table 1 shows the line data for an expanded form of Garver's 6-bus test system, the bus data of which are given in Table 2. The per unit (pu) values in Table 1 are computed considering an 100-MVA power base. The reference bus is bus 1.

The students are asked to solve the DC OPF problem using MATLAB Optimization Toolbox; to determine the contribution of generators and loads to transmission usage based on distribution factors tracing method; and to compute the cost

**Table 1** Line Data for Garver's 6-Bus Test System

From bus $l$	To bus $k$	Line reactance, $x_{lk}$ (pu)	Line length (km)	Line capacity, $P_{B,max}$ (MW)	Annualized cost of investment, $TIC_{lk}$ (k€)
1	2	0.40	40	100	40
1	4	0.60	60	80	60
1	5	0.20	20	100	20
2	3	0.20	20	100	20
2	4	0.40	40	100	40
3	5	0.10	20	200	40
2	6	0.15	30	200	60
4	6	0.15	30	200	60

**Table 2** Bus Data for Garver's 6-Bus Test System

Bus $k$	Minimum production, $P_{g,min}$ (MW)	Maximum production, $P_{g,max}$ (MW)	Load, $P_L$ (MW)	Generator bid, $q_k$ (€/MWh)
1	0	150	80	10
2	0	0	240	0
3	0	360	40	20
4	0	0	160	0
5	0	0	240	0
6	0	600	0	30

allocation to generators and loads using the MW-mile original pricing method by assuming a 30–70% share of the transmission costs between generators and loads.

### DC OPF Problem Formulation

The DC OPF problem is formulated as follows [11]:

$$\min f = \sum_k q_k P_{gk} + G \sum_k r_k \quad (9)$$

subject to:

$$\mathbf{P}_B = \mathbf{D} \cdot \mathbf{A} \cdot \boldsymbol{\Theta} \quad (10)$$

$$\mathbf{B} \cdot \boldsymbol{\Theta} - \mathbf{P}_G - \mathbf{r} = -\mathbf{P}_L \quad (11)$$

$$\mathbf{P}_{g,\min} \leq \mathbf{P}_G \leq \mathbf{P}_{g,\max} \quad (12)$$

$$-\mathbf{P}_{B,\max} \leq \mathbf{P}_B \leq \mathbf{P}_{B,\max} \quad (13)$$

$$-\pi \leq \boldsymbol{\Theta} \leq \pi \quad (14)$$

$$\mathbf{0} \leq \mathbf{r} \leq \mathbf{P}_L \quad (15)$$

where  $N$  is the number of buses,  $M$  is the number of transmission lines,  $q_k$  is the generation bid of bus  $k$ ,  $P_{gk}$  is the generation of bus  $k$ ,  $G$  a penalty term to prevent power not supplied (in this example we assume  $G = 1,000$  €/h),  $r_k$  the load not served of bus  $k$ ,  $\mathbf{P}_B = [P_{b1} \ P_{b2} \ \dots \ P_{bM}]^T$  the vector of line flows,  $\mathbf{P}_{B,\max}$  the vector of maximum line flows,  $\mathbf{P}_G = [P_{g1} \ P_{g2} \ \dots \ P_{gN}]^T$  the vector of bus generations,  $\boldsymbol{\Theta} = [\theta_1 \ \theta_2 \ \dots \ \theta_N]^T$  the vector of bus angles in radians,  $\mathbf{r} = [r_1 \ r_2 \ \dots \ r_N]^T$  the vector of load not served,  $\mathbf{P}_L$  the vector of bus loads.  $\mathbf{B}$  is the  $N \times N$  bus admittance matrix, and  $\mathbf{A}$  is the  $M \times N$  network incidence matrix [21].  $\mathbf{D}$  is an  $M \times M$  matrix with zero nondiagonal elements, while its diagonal elements are computed by  $d_{mm} = 1/x_{lk}$ , where  $x_{lk}$  is the reactance of line  $l-k$ .

### Network Matrices

The network matrices  $\mathbf{A}$ ,  $\mathbf{D}$ , and  $\mathbf{B}$  for the considered 6-bus test system are as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 2.5000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.6667 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6.6667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.6667 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 9.1667 & -2.5000 & 0 & -1.6667 & -5.000 & 0 \\ -2.5000 & 16.6667 & -5.0000 & -2.5000 & 0 & -6.6667 \\ 0 & -5.0000 & 15.0000 & 0 & -10.0000 & 0 \\ -1.6667 & -2.5000 & 0 & 10.8333 & 0 & -6.6667 \\ -5.0000 & 0 & -10.0000 & 0 & 15.0000 & 0 \\ 0 & -6.6667 & 0 & -6.6667 & 0 & 13.3333 \end{bmatrix}$$

### DC OPF Formulation for the 6-Bus Test System

The vector of design variables is as follows:

$$\mathbf{X} = [\mathbf{P}_G \ \mathbf{P}_B \ \boldsymbol{\Theta} \ \mathbf{r}]^T = [P_{g1} \ P_{g2} \ P_{g3} \ P_{g4} \ P_{g5} \ P_{g6} \ P_{b1} \ P_{b2} \ P_{b3} \ P_{b4} \ P_{b5} \ P_{b6} \ P_{b7} \ P_{b8} \ \theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_5 \ \theta_6 \ r_1 \ r_2 \ r_3 \ r_4 \ r_5 \ r_6]^T$$

where vectors  $\mathbf{P}_G$ ,  $\mathbf{P}_B$ , and  $\mathbf{r}$  are in pu, while vector  $\boldsymbol{\Theta}$  is in radians.

The DC OPF problem for the 6-bus test system is formulated as follows:

$$\min_{\mathbf{X}} f = \min_{\mathbf{X}} [10P_{g1} + 20P_{g3} + 30P_{g6} + 1,000(r_1 + r_2 + r_3 + r_4 + r_5 + r_6)] \quad (16)$$

$$P_{b1} = 2.5\theta_1 - 2.5\theta_2 \quad P_{b2} = 1.6667\theta_1 - 1.6667\theta_4 \quad P_{b3} = 5\theta_1 - 5\theta_5 \quad (17)$$

$$P_{b4} = 5\theta_2 - 5\theta_3 \quad P_{b5} = 2.5\theta_2 - 2.5\theta_4 \quad P_{b6} = 10\theta_3 - 10\theta_5 \quad (18)$$

$$P_{b7} = 6.6667\theta_2 - 6.6667\theta_6 \quad P_{b8} = 6.6667\theta_4 - 6.6667\theta_6 \quad (19)$$

$$(9.1667\theta_1 - 2.5\theta_2 - 1.6667\theta_4 - 5\theta_5) - P_{g1} - r_1 = -0.8 \quad (20)$$

$$(-2.5\theta_1 + 16.6667\theta_2 - 5\theta_3 - 2.5\theta_4 - 6.6667\theta_6) - P_{g2} - r_2 = -2.4 \quad (21)$$

$$(-5\theta_2 + 15\theta_3 - 10\theta_5) - P_{g3} - r_3 = -0.4 \quad (22)$$

$$(-1.6667\theta_1 - 2.5\theta_2 + 10.8333\theta_4 - 6.6667\theta_6) - P_{g4} - r_4 = -1.6 \quad (23)$$

$$(-5\theta_1 - 10\theta_3 + 15\theta_5) - P_{g5} - r_5 = -2.4 \quad (24)$$

$$(-6.6667\theta_2 - 6.6667\theta_4 + 13.3333\theta_6) - P_{g6} - r_6 = 0.0 \quad (25)$$

$$0 \leq P_{g1} \leq 1.5 \quad 0 \leq P_{g2} \leq 0 \quad 0 \leq P_{g3} \leq 3.6 \quad 0 \leq P_{g4} \leq 0 \quad 0 \leq P_{g5} \leq 0 \quad 0 \leq P_{g6} \leq 6 \quad (26)$$

$$-1 \leq P_{b1} \leq 1 \quad -0.8 \leq P_{b2} \leq 0.8 \quad -1 \leq P_{b3} \leq 1 \quad -1 \leq P_{b4} \leq 1 \quad (27)$$

$$-1 \leq P_{b5} \leq 1 \quad -2 \leq P_{b6} \leq 2 \quad -2 \leq P_{b7} \leq 2 \quad -2 \leq P_{b8} \leq 2 \quad (28)$$

$$0 \leq \theta_1 \leq 0 \quad -\pi \leq \theta_2 \leq \pi \quad -\pi \leq \theta_3 \leq \pi \quad -\pi \leq \theta_4 \leq \pi \quad -\pi \leq \theta_5 \leq \pi \quad -\pi \leq \theta_6 \leq \pi \quad (29)$$

$$0 \leq r_1 \leq 0.8 \quad 0 \leq r_2 \leq 2.4 \quad 0 \leq r_3 \leq 0.4 \quad 0 \leq r_4 \leq 1.6 \quad 0 \leq r_5 \leq 2.4 \quad 0 \leq r_6 \leq 0 \quad (30)$$

### DC OPF Solution Using MATLAB

The linear programming problem of (16)–(30) can be solved using the function *linprog* of MATLAB Optimization Toolbox. Details about function *linprog* and how to call it can be found in Ref. [22]. The MATLAB code to solve the optimization problem (16)–(30) is the following:

```
Aeq(:,1:14)=[
    0    0    0    0    0    0   -1    0    0    0    0    0    0    0
    0    0    0    0    0    0    0   -1    0    0    0    0    0    0
    0    0    0    0    0    0    0    0   -1    0    0    0    0    0
    0    0    0    0    0    0    0    0    0   -1    0    0    0    0
    0    0    0    0    0    0    0    0    0    0   -1    0    0    0
    0    0    0    0    0    0    0    0    0    0    0   -1    0    0
    0    0    0    0    0    0    0    0    0    0    0    0   -1    0
   -1    0    0    0    0    0    0    0    0    0    0    0    0    0
    0   -1    0    0    0    0    0    0    0    0    0    0    0    0
    0    0   -1    0    0    0    0    0    0    0    0    0    0    0
    0    0    0   -1    0    0    0    0    0    0    0    0    0    0
    0    0    0    0   -1    0    0    0    0    0    0    0    0    0
    0    0    0    0    0   -1    0    0    0    0    0    0    0    0];

Aeq(:,15:20)=[
    2.5000   -2.5000         0         0         0         0
    1.6667         0         0   -1.6667         0         0
    5.0000         0         0         0   -5.0000         0
         0    5.0000   -5.0000         0         0         0
         0    2.5000         0   -2.5000         0         0
         0         0   10.0000         0  -10.0000         0
         0    6.6667         0         0         0   -6.6667
         0         0         0    6.6667         0   -6.6667
    9.1667   -2.5000         0   -1.6667   -5.0000         0
   -2.5000   16.6667   -5.0000   -2.5000         0   -6.6667
         0   -5.0000   15.0000         0  -10.0000         0
   -1.6667   -2.5000         0   10.8333         0   -6.6667
   -5.0000         0  -10.0000         0   15.0000         0
         0   -6.6667         0   -6.6667         0   13.3333];
```

```

Aeq(:,21:26)=[
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    0    0    0    0    0    0
    -1   0    0    0    0    0
    0   -1   0    0    0    0
    0    0   -1   0    0    0
    0    0    0   -1   0    0
    0    0    0    0   -1   0
    0    0    0    0    0   -1];
beq=[0 0 0 0 0 0 0 -0.8000 -2.4000 -0.4000 -1.6000 -2.4000 0]';
LB= [0 0 0 0 0 0 -1 -0.8 -1 -1 -1 -2 -2 -2 0 -pi -pi -pi -pi -pi 0 0 0 0 0]';
UB= [1.5 0 3.6 0 0 6 1 0.8 1 1 1 2 2 2 0 pi pi pi pi pi 0.8 2.4 0.4 1.6 2.4 0]';
A=[]; b=[]; x0=[];
c = [1000 0 2000 0 0 3000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 100000 100000 100000 100000 100000 100000]';
options = optimset('LargeScale', 'off');
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]=linprog(c,A,b,Aeq,beq,LB,UB,x0,options);

```

After the execution of the above MATLAB script file, the solution to the optimization problem (16)–(30) is stored in the output variable  $X$  of function *linprog*. More specifically, the following results are obtained:

$$\mathbf{X} = [\mathbf{P}_G \quad \mathbf{P}_B \quad \Theta \quad \mathbf{r}]^T$$

where

$$\mathbf{P}_G = [1.5000 \quad 0.0 \quad 3.331915 \quad 0.0 \quad 0.0 \quad 2.768085]^T$$

$$\mathbf{P}_B = [0.165957 \quad 0.134043 \quad 0.4 \quad -0.931915 \quad 0.035106 \quad 2.0 \quad -1.337234 \quad -1.430851]^T$$

$$\Theta = [0.0 \quad -0.0664 \quad 0.12 \quad -0.0804 \quad -0.08 \quad 0.1342]^T$$

$$\mathbf{r} = [0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0]^T$$

### Generation Shift Distribution Factors (GSDF)

The generation shift distribution factor,  $\text{GSDF}_{lk,i}$ , which represents the sensitivity of power flow in line  $l-k$  with respect to injection (i.e., generation minus load) in bus  $i$ , is computed by [18]

$$\text{GSDF}_{lk,i} = \frac{S_{li} - S_{ki}}{x_{lk}} \quad (31)$$

where  $x_{lk}$  is the reactance of line  $l-k$ , while the matrix  $S$  is calculated as follows:

$$S = \begin{bmatrix} 0 & 0 \\ 0 & B_{rd}^{-1} \end{bmatrix} \quad (32)$$

where  $B_{rd}$  is a copy of bus admittance matrix  $B$  ignoring its column and row associated with the reference bus. In the 6-bus test system, the reference bus is bus 1, so:

$$B_{rd}^{-1} = \begin{bmatrix} 16.6667 & -5.0000 & -2.5000 & 0 & -6.6667 \\ -5.0000 & 15.0000 & 0 & -10.0000 & 0 \\ -2.5000 & 0 & 10.8333 & 0 & -6.6667 \\ 0 & -10.0000 & 0 & 15.0000 & 0 \\ -6.6667 & 0 & -6.6667 & 0 & 13.3333 \end{bmatrix}^{-1} = \begin{bmatrix} 0.1725 & 0.1035 & 0.1342 & 0.0690 & 0.1534 \\ 0.1035 & 0.1821 & 0.0805 & 0.1214 & 0.0920 \\ 0.1342 & 0.0805 & 0.2377 & 0.0537 & 0.1859 \\ 0.0690 & 0.1214 & 0.0537 & 0.1476 & 0.0613 \\ 0.1534 & 0.0920 & 0.1859 & 0.0613 & 0.2446 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 \\ 0 & B_{rd}^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1725 & 0.1035 & 0.1342 & 0.0690 & 0.1534 \\ 0 & 0.1035 & 0.1821 & 0.0805 & 0.1214 & 0.0920 \\ 0 & 0.1342 & 0.0805 & 0.2377 & 0.0537 & 0.1859 \\ 0 & 0.0690 & 0.1214 & 0.0537 & 0.1476 & 0.0613 \\ 0 & 0.1534 & 0.0920 & 0.1859 & 0.0613 & 0.2446 \end{bmatrix}$$

For example, the generation shift distribution factor,  $\text{GSDF}_{24,3}$ , of line 2–4 with respect to injection in bus 3, is

$$\text{GSDF}_{24,3} = \frac{S_{23} - S_{43}}{x_{24}} = \frac{0.1035 - 0.0805}{0.4} = 0.0575$$

The values of all the generation shift distribution factors are presented in Table 3.

### Generalized Generation Distribution Factors (GGDF)

The generalized generation distribution factor,  $\text{GGDF}_{lk,i}$  which represents the portion of generation supplied by generator  $i$  that flows in line  $l$ – $k$ , is computed by [18]

$$\text{GGDF}_{lk,rb} = \frac{F_{lk}^0 - \sum_{i=1, i \neq rb}^N \text{GSDF}_{lk,i} P_{gi}}{\sum_{i=1}^N P_{gi}} \quad (33)$$

$$\text{GGDF}_{lk,i} = \text{GGDF}_{lk,rb} + \text{GSDF}_{lk,i} \quad (34)$$

where  $F_{lk}^0$  is the power flow in line  $l$ – $k$  as is computed by DC OPF, while the index  $rb$  refers to the reference bus, so (33) computes the GGDF for the reference bus, while (34) computes the GGDF for the other generator buses.

For example, in the 6-bus test system, we have

$$\begin{aligned} \text{GGDF}_{12,1} &= \frac{F_{12}^0 - \sum_{i=2}^N \text{GSDF}_{12,i} P_{gi}}{\sum_{i=1}^N P_{gi}} \\ &= \frac{0.165957 - (-0.2588 \times 3.331915 - 0.3834 \times 2.768085)}{1.5 + 3.331915 + 2.768085} \\ &= 0.2749 \end{aligned}$$

$$\text{GGDF}_{12,3} = \text{GGDF}_{12,1} + \text{GSDF}_{12,3} = 0.2749 - 0.2588 = 0.0161$$

The values of all GGDF are presented in Table 4, where G1 denotes the generator of bus 1.

### Transmission Usage Allocation to Generators Using GGDF

The allocation of the usage of transmission line  $l$ – $k$  to generator  $i$  is computed by [18]

$$\text{FG}_{lk,i} = \text{GGDF}_{lk,i} P_{gi} \quad (35)$$

For example, in the 6-bus test system, we have

$$\begin{aligned} \text{FG}_{12,G3} &= \text{GGDF}_{12,G3} \times P_{g3} \\ &= 0.0161 \times (3.331915 \text{ pu}) \times (100 \text{ MVA}) \\ &= 5.3786 \text{ MW} \end{aligned}$$

The allocation of the usage of transmission lines to generators is presented in Table 5.

### Generalized Load Distribution Factors (GLDF)

The generalized load distribution factor,  $\text{GLDF}_{lk,i}$ , which determines the contribution of each load  $i$  to line flows in each line  $l$ – $k$ , is computed by [18]

$$\text{GLDF}_{lk,rb} = \frac{F_{lk}^0 + \sum_{i=1, i \neq rb}^N \text{GSDF}_{lk,i} P_{Li}}{\sum_{i=1}^N P_{Li}} \quad (36)$$

$$\text{GLDF}_{lk,i} = \text{GLDF}_{lk,rb} - \text{GSDF}_{lk,i} \quad (37)$$

where  $F_{lk}^0$  is the power flow in line  $l$ – $k$  as is computed by DC OPF,  $P_{Li}$  is the load at bus  $i$ , while the index  $rb$  refers to the

**Table 3** GSDF With Respect to the Reference Bus 1 for Garver's 6-Bus Test System

From bus	To bus	GSDF for each bus					
		Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
1	2	0	−0.4313	−0.2588	−0.3355	−0.1725	−0.3834
1	4	0	−0.2236	−0.1342	−0.3962	−0.0895	−0.3099
1	5	0	−0.3450	−0.6070	−0.2684	−0.7380	−0.3067
2	3	0	0.3450	−0.3930	0.2684	−0.2620	0.3067
2	4	0	0.0958	0.0575	−0.2588	0.0383	−0.0815
3	5	0	0.3450	0.6070	0.2684	−0.2620	0.3067
2	6	0	0.1278	0.0767	−0.3450	0.0511	−0.6086
4	6	0	−0.1278	−0.0767	0.3450	−0.0511	−0.3914



**Table 4** GGDF for Garver's 6-Bus Test System

From bus	To bus	GGDF for each generator		
		G1	G3	G6
1	2	0.2749	0.0161	-0.1085
1	4	0.1893	0.0552	-0.1206
1	5	0.4305	-0.1766	0.1238
3	2	0.0620	0.4550	-0.2447
2	4	0.0091	0.0666	-0.0724
3	5	-0.1147	0.4923	0.1920
6	2	-0.0121	-0.0888	0.5965
6	4	0.0121	0.0888	0.4035

**Table 5** Allocation of Transmission Usage to Generators for Garver's 6-Bus Test System

From bus	To bus	FG (MW) for each generator			Total (MW)
		G1	G3	G6	
1	2	41.2393	5.3786	-30.0221	16.5957
1	4	28.4009	18.3768	-33.3734	13.4043
1	5	64.5704	-58.8282	34.2578	40.0000
3	2	9.3072	151.6086	-67.7243	93.1915
2	4	1.3620	22.1866	-20.0380	3.5106
3	5	-17.2020	164.0465	53.1554	200.0000
6	2	-1.8160	-29.5822	165.1216	133.7234
6	4	1.8160	29.5822	111.6869	143.0851

reference bus, so (36) computes the GLDF for the reference bus, while (37) computes the GLDF for the other load buses.

For example, in the 6-bus test system, we have

$$GLDF_{12,1} = \frac{0.165957 \times 100 + (-0.4313 \times 240 - 0.2588 \times 40 - 0.3355 \times 160 - 0.1725 \times 240)}{80 + 240 + 40 + 160 + 240} = -0.2531$$

$$GLDF_{12,5} = GLDF_{12,1} - GSDF_{12,5} = -0.2531 - (-0.1725) = -0.0806$$

The values of all GLDF are presented in Table 6.

#### Transmission Usage Allocation to Loads Using GLDF

The allocation of the usage of transmission line  $l-k$  to load  $i$  is computed by [18]

$$FL_{lk,i} = GLDF_{lk,i} P_{Li} \quad (38)$$

**Table 7** Allocation of Transmission Usage to Loads for Garver's 6-bus Test System

From bus	To bus	FL (MW) for each load					Total (MW)
		L1	L2	L3	L4	L5	
1	2	-20.2474	42.7723	0.2278	13.1794	-19.3363	16.5957
1	4	-13.7362	12.4656	-1.5007	35.9143	-19.7388	13.4043
1	5	-30.2270	-7.8695	9.1676	-17.5147	86.4436	40.0000
3	2	4.8458	97.3489	-13.2960	52.6309	-48.3380	93.1915
2	4	-0.3569	-24.0738	-2.4788	40.6920	-10.2719	3.5106
3	5	30.2270	7.8695	-9.1676	17.5147	153.5564	200.0000
6	2	15.0447	75.8050	10.5894	-25.1183	57.4025	133.7234
6	4	14.0930	11.6082	3.9794	83.3937	30.0107	143.0851

**Table 6** GLDF With Respect to the Reference Bus 1 for Garver's 6-Bus Test System

From bus	To bus	GLDF for each load				
		L1	L2	L3	L4	L5
1	2	-0.2531	0.1782	0.0057	0.0824	-0.0806
1	4	-0.1717	0.0519	-0.0375	0.2245	-0.0822
1	5	-0.3778	-0.0328	0.2292	-0.1095	0.3602
3	2	0.0606	0.4056	-0.3324	0.3289	-0.2014
2	4	-0.0045	-0.1003	-0.0620	0.2543	-0.0428
3	5	0.3778	0.0328	-0.2292	0.1095	0.6398
6	2	0.1881	0.3159	0.2647	-0.1570	0.2392
6	4	0.1762	0.0484	0.0995	0.5212	0.1250

For example, in the 6-bus test system, we have

$$FL_{12,L3} = GLDF_{12,L3} P_{L3} = 0.0057 \times 40 = 0.2278 \text{ MW}$$

The allocation of the usage of transmission lines to loads is presented in Table 7, where L1 denotes the load of bus 1.

#### Charge of Generators and Loads Using Original MW-Mile Method

The total annualized transmission investment cost is 340 k€, as can be seen from Table 1. Thirty percent of this cost is charged to generators and the rest 70% is charged to loads, which means that generators will pay 102 k€ and loads will pay 238 k€, that is,  $TC_G = 102 \text{ k€}$  and  $TC_L = 238 \text{ k€}$ .

The charge  $TIC_{12}|FG_{12,G1}|$  of generator 1 due to power flow in line 1-2 is computed as follows:

$$TIC_{12}|FG_{12,G1}| = 40 \times |41.2393| = 1649.5712 \text{ k€}$$

where  $TIC_{12}$  is the annualized cost of investment for line 1-2.

Similarly, the charge  $TIC_{lk}|F_{lk,Gi}|$  for each generator  $Gi$  at bus  $i$  due to the power flow in each transmission line  $l-k$  is computed and the results are shown in Table 8.

**Table 8** Computation of the Total Charge (k€) Per Generator for Garver's 6-Bus Test System, Considering That the Generators Will Pay 102 k€, Which Corresponds to 30% of the Total Annualized Transmission Investment Cost

From bus	To bus	TIC <sub>lk</sub> (k€)	Charge multiplied with line flow per generator		
			TIC <sub>lk</sub>  FG <sub>lk,G1</sub>	TIC <sub>lk</sub> ·  FG <sub>lk,G3</sub>	TIC <sub>lk</sub> ·  FG <sub>lk,G6</sub>
1	2	40	1649.5712	215.1430	1200.8844
1	4	60	1704.0525	1102.6078	2002.4049
1	5	20	1291.4074	1176.5632	685.1558
3	2	20	186.1443	3032.1714	1354.4859
2	4	40	54.4813	887.4648	801.5205
3	5	40	688.0780	6561.8610	2126.2170
6	2	60	108.9625	1774.9296	9907.2964
6	4	60	108.9625	1774.9296	6701.2143
Total		340	5791.6597	16525.6704	24779.1792
$\sum_{Gi \in T} \sum_{lk \in K} TIC_{lk}  FG_{lk,Gi} $			47096.5092		
Total charge per generator, TCG <sub>Gi</sub> (k€)			12.5434	35.7907	53.6659

The total charge (k€) of generator 1 is computed by (2) as follows:

$$TCG_{G1} = TC_G \frac{\sum_{lk \in K} TIC_{lk} |FG_{lk,G1}|}{\sum_{Gi \in T} \sum_{lk \in K} TIC_{lk} |FG_{lk,Gi}|}$$

$$= 102 \times \frac{5791.6597}{47096.5092} = 12.5434 \text{ k}$$

where  $K$  is the set of the eight transmission lines and  $T$  is the set of the three generators.

Similarly, the total charge per load is computed and the results are presented in Table 9, where  $L$  denotes the set of the five loads.

### Students Feedback

The above educational example has been used during the last 3 years at the National Technical University of Athens (NTUA) to help teaching the undergraduate course of power system economics. This educational example, which is distributed to the students at the beginning of the semester, includes detailed

presentation of the calculations involved together with MATLAB script files that easily perform the necessary computations. Here, due to space limitations, a brief presentation of the educational example was adopted.

The instructor presents in the class all the necessary theory of transmission pricing, together with the arithmetic example. As a first laboratory exercise, the students solve the educational example in a lab with 48 computers. Groups of 48 students are formed and each student uses one of the computers of the laboratory for solving the transmission pricing educational example. The instructor together with four advanced postgraduate students facilitate the students to solve the educational example. As a second laboratory exercise, the students are educated to use the transmission pricing software that is presented in the next section.

The Electrical and Computer Engineering students at NTUA are well familiar with computers and MATLAB at their sixth semester of studies, when the power system economics course is taught. Students indicated that the above educational example allows them to clearly understand the transmission pricing issues.

**Table 9** Computation of the Total Charge (k€) Per Load for Garver's 6-Bus Test System, Considering That the Loads Will Pay 238 k€, Which Corresponds to 70% of the Total Annualized Transmission Investment Cost

From bus	To bus	TIC <sub>lk</sub> (k€)	Charge multiplied with line flow per load				
			TIC <sub>lk</sub>  FL <sub>lk,L1</sub>	TIC <sub>lk</sub>  FL <sub>lk,L2</sub>	TIC <sub>lk</sub>  FL <sub>lk,L3</sub>	TIC <sub>lk</sub>  FL <sub>lk,L4</sub>	TIC <sub>lk</sub>  FL <sub>lk,L5</sub>
1	2	40	809.8945	1710.8916	9.1103	527.1759	773.4534
1	4	60	824.1695	747.9387	90.0400	2154.8558	1184.3297
1	5	20	604.5401	157.3903	183.3530	350.2943	1728.8717
3	2	20	96.9157	1946.9770	265.9192	1052.6173	966.7610
2	4	40	14.2750	962.9529	99.1503	1627.6800	410.8762
3	5	40	1209.0802	314.7806	366.7059	700.5885	6142.2566
6	2	60	902.6822	4548.3022	635.3667	1507.0957	3444.1488
6	4	60	845.5821	696.4906	238.7655	5003.6242	1800.6440
Total		340	5307.1393	11085.7239	1888.4107	12923.9316	16451.3415
$\sum_{Li \in L} \sum_{lk \in K} TIC_{lk}  FL_{lk,Li} $			47656.5470				
Total charge per load, TCL <sub>Li</sub> (k€)			26.5042	55.3629	9.4309	64.5430	82.1591

They appreciated their active involvement, since they solve almost alone the complete transmission-pricing problem, from the beginning to the end. The experience they gain allows them to easily solve their exercises and to perform better at the exams of the course. It is also important that the interest of students for this course was increased thanks to the introduction of computer-aided education methodology.

## TRANSMISSION PRICING SOFTWARE

### Programming Environment

The transmission pricing software (TPS) was developed using MATLAB programming language because of the following main reasons:

- (1) MATLAB is a powerful programming environment. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation [3].
- (2) MATLAB provides a set of tools for easily creating GUIs. As the TPS is designed for power engineering education, the GUI is absolutely necessary because it helps the student arrive at the final solution by

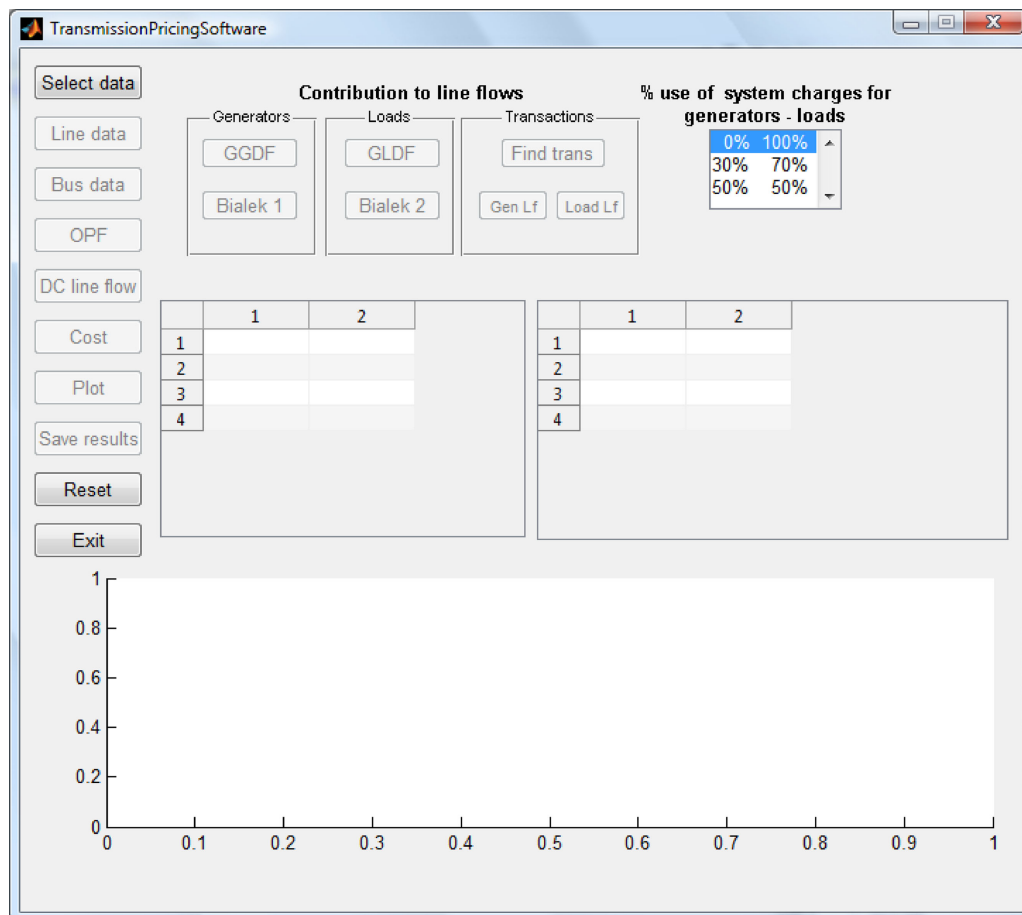
visualizing each step of the design process [4]. Moreover, the GUI makes things simple for the end-users of the program [5].

- (3) The MATLAB functions that have been developed for TPS can be extended and used for future research.

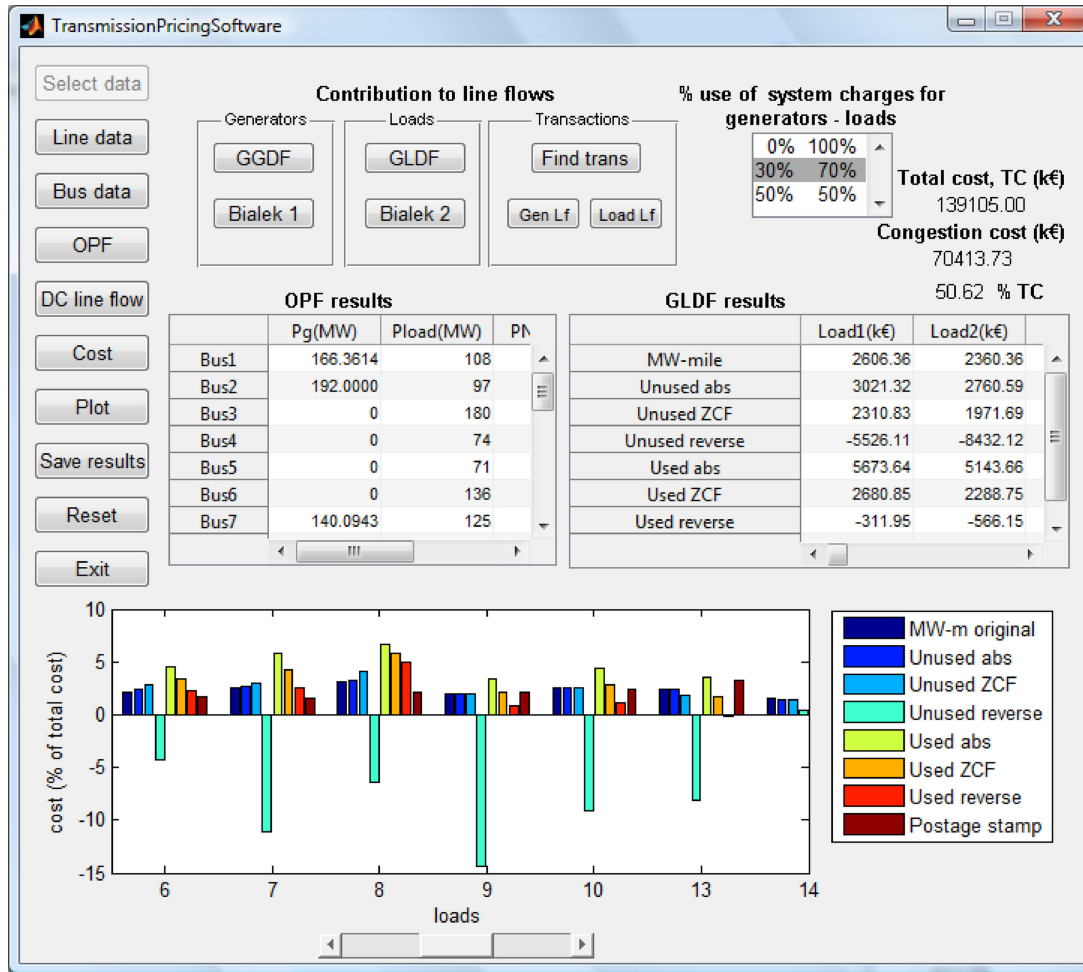
### Graphical User Interface

In TPS, the following three tracing methods are implemented: distribution factors, Bialek, and the minimum power distance method. In addition, TPS implements eight transmission pricing methods: MW-mile original, unused absolute MW-mile, unused zero counter flow MW-mile, unused reverse MW-mile, used absolute MW-mile, used zero counter flow MW-mile, used reverse MW-mile, and postage stamp method.

The GUI of TPS has been designed so as the students can easily define and solve transmission pricing problems. The main reason GUI is used is because it makes things simple for the end-users of the program [5]. More specifically, the GUI of TPS consists of one main window that is shown in Figure 1. The student can take advantage of the full functionality of TPS using 17 different push buttons: "Select data," "Line data," "Bus data," "OPF," "DC line flow," "Cost," "Plot," "Save results," "Reset," "Exit," "GGDF," "Bialek 1," "GLDF," "Bialek 2," "Find trans," "Gen Lf," "Load Lf," "Cost," "Plot," "Save results," "Reset," and "Exit" (see Fig. 1).



**Figure 1** Main window of TPS GUI. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]



**Figure 2** Indicative results for IEEE RTS 24-bus system. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

As can be seen in Figure 2, the results of TPS are presented in two forms: (a) in the two tables in the middle of the GUI, and (b) in the graph at the bottom of the GUI.

The TPS algorithm is executed through the following four main steps:

- Step 1: Data input.
- Step 2: Calculation of transmission line flows.
- Step 3: Calculation of the contribution of network users to line flows.
- Step 4: Computation of charges for network users.

These steps will be analytically described in the following.

### Step 1: Data Input

In TPS, all the necessary data are inserted in the form of a Microsoft Excel file. More specifically, by clicking the button "Select data" (Fig. 1), a dialog box opens, in which the student can choose the Excel file that contains the data of the transmission pricing problem.

The Excel file is composed of two worksheets. The first worksheet contains the transmission line data, that is, for each

transmission line it contains the sending bus, the receiving bus, the line reactance in per unit (pu), the line length (km), the line capacity (MW), and the annualized cost (k€) of investment for the transmission line. The second worksheet contains the bus data, that is, for each bus it contains its load (MW) for each time period, its production (MW), as well as the following additional data for solving the OPF problem: minimum production (MW), maximum production (MW), and generator bid (€/MWh).

Next, the student can see the selected input data for network buses and lines in the two tables of the GUI by pressing the buttons "Bus data" and "Line data", respectively.

Next, the students select the percentage of charge for generators and loads from the list box at the top right of the GUI (Fig. 1) among the three available options: (a) 0–100%, (b) 30–70%, and (c) 50–50% cost allocation between generators and loads.

### Step 2: Calculation of Transmission Line Flows

Initially, the student presses the button "OPF" in order to solve the OPF problem. The results of the OPF are displayed in the two tables of the GUI. More specifically, the first table presents

the following results for the system buses: the production (MW), the load served (MW), the power not supplied (MW), and the LMP (€/MWh). On the other hand, the second table presents for each transmission line its line flow (MW) and its congestion cost (k€). Moreover, the total congestion cost for all lines is computed and displayed in k€ as well as a percentage of the total cost (TC), as shown in Figure 2.

### Step 3: Calculation of the Contribution of Network Users to Line Flows

The student can use three alternative methods for computing the contribution of generators to line flows:

- (1) GGDF method by pressing the button “GGDF.”
- (2) Bialek upstream method by pressing the button “Bialek 1.”
- (3) Minimum power distance method by first pressing the button “Find trans” and next by pressing the button “Gen Lf.”

Three alternative methods can be used for computing the contribution of loads to line flows:

- (1) GLDF method by pressing the button “GLDF.”
- (2) Bialek downstream method by pressing the button “Bialek 2.”
- (3) Minimum power distance method by first pressing the button “Find trans” and next by pressing the button “Load Lf.”

### Step 4: Computation of Charges for Network Users

The student presses the button “Cost” and as a result the transmission charges of generators or loads to the line flows will be calculated (using eight different transmission pricing methods) and displayed in the second table of the GUI. For example, if during step 3 the student has pressed the button “GLDF” and during step 4 has pressed the button “Cost”, then during step 4 the charges of loads will be presented in the second table of GUI (see Fig. 2), where the contribution of loads to line flows was computed based on GLDF method, while the charge of loads was computed by eight different transmission pricing methods:

- (1) MW-mile original (abbreviated in TPS as *MW-m original*).
- (2) Unused absolute MW-mile (abbreviated in TPS as *unused abs*).
- (3) Unused zero counter flow MW-mile (abbreviated in TPS as *unused ZCF*).
- (4) Unused reverse MW-mile (abbreviated in TPS as *unused reverse*).
- (5) Used absolute MW-mile (abbreviated in TPS as *used abs*).
- (6) Used zero counter flow MW-mile (abbreviated in TPS as *used ZCF*).
- (7) Used reverse MW-mile (abbreviated in TPS as *used reverse*).
- (8) Postage stamp.

**Table 10** Generators Contribution to Line Flows on Garver's 6-Bus Test System Using GGDF, Bialek Upstream and Minimum Power Distance Tracing Method

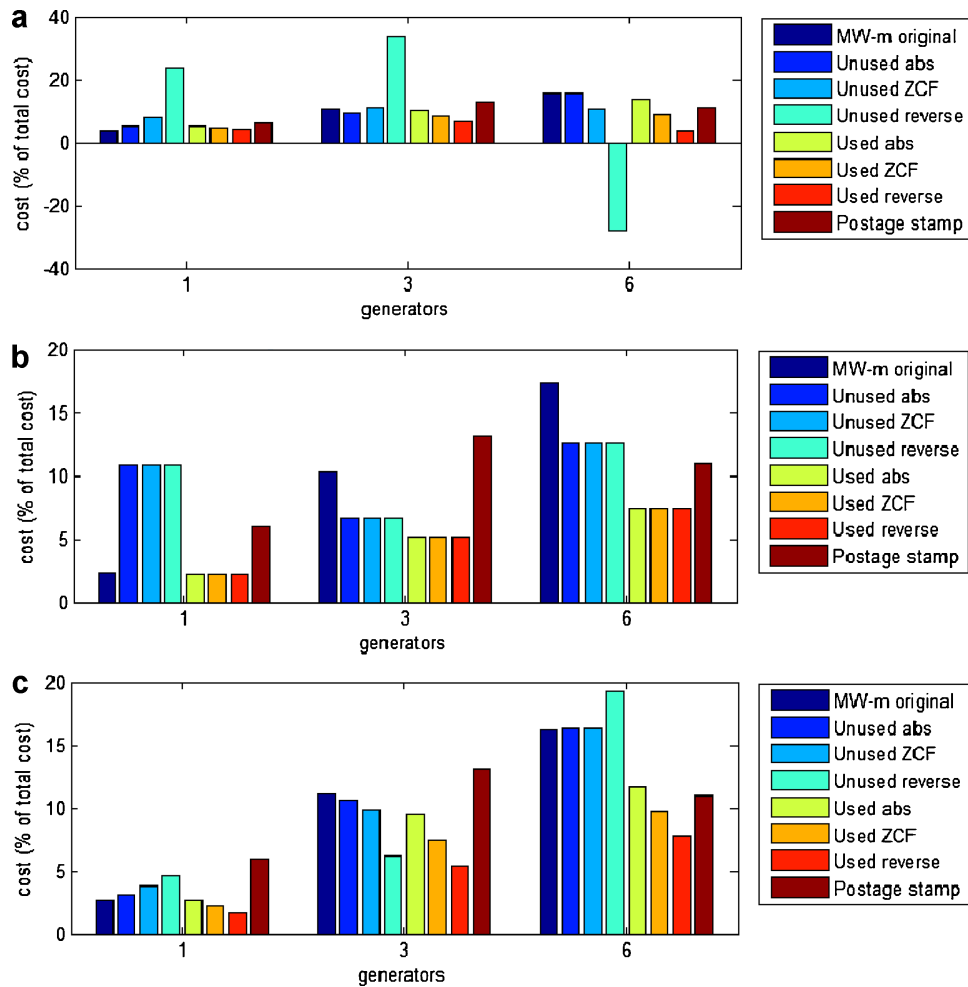
Lines			GGDF			Bialek upstream			Minimum power distance		
From bus	To bus	Line flow (MW)	G1 (MW)	G3 (MW)	G6 (MW)	G1 (MW)	G3 (MW)	G6 (MW)	G1 (MW)	G3 (MW)	G6 (MW)
1	2	16.60	41.24	5.38	−30.02	16.60	0.00	0.00	12.08	6.59	−2.07
1	4	13.40	28.40	18.38	−33.37	13.40	0.00	0.00	6.26	3.42	3.73
1	5	40.00	64.57	−58.83	34.26	40.00	0.00	0.00	51.66	−10.01	−1.66
3	2	93.19	9.31	151.61	−67.72	0.00	93.19	0.00	−18.34	113.19	−1.66
2	4	3.51	1.36	22.19	−20.04	0.24	1.34	1.93	−2.68	−1.46	7.66
3	5	200.00	−17.20	164.05	53.16	0.00	200.00	0.00	18.34	180.01	1.66
6	2	133.72	−1.82	−29.58	165.12	0.00	0.00	133.72	3.58	1.95	128.19
6	4	143.09	1.82	29.58	111.69	0.00	0.00	143.09	−3.58	−1.95	148.62

**Table 11** Loads Contribution to Line Flows on Garver's 6-Bus Test System Using GLDF, Bialek Downstream, and Minimum Power Distance Tracing Method

Lines			GLDF					Bialek downstream					Minimum power distance				
From bus	To bus	Line flow (MW)	L1 (MW)	L2 (MW)	L3 (MW)	L4 (MW)	L5 (MW)	L1 (MW)	L2 (MW)	L3 (MW)	L4 (MW)	L5 (MW)	L1 (MW)	L2 (MW)	L3 (MW)	L4 (MW)	L5 (MW)
1	2	16.60	−20.25	42.77	0.23	13.18	−19.34	0.00	16.36	0.00	0.24	0.00	0.00	26.85	0.00	−7.67	−2.59
1	4	13.40	−13.74	12.47	−1.50	35.91	−19.74	0.00	0.00	0.00	13.40	0.00	0.00	0.94	0.00	13.80	−1.34
1	5	40.00	−30.23	−7.87	9.17	−17.51	86.44	0.00	0.00	0.00	0.00	40.00	0.00	−27.80	0.00	−6.13	73.93
3	2	93.19	4.85	97.35	−13.30	52.63	−48.34	0.00	91.85	0.00	1.34	0.00	0.00	95.40	0.00	−6.13	3.93
2	4	3.51	−0.36	−24.07	−2.48	40.69	−10.27	0.00	0.00	0.00	3.51	0.00	0.00	−25.44	0.00	28.37	0.58
3	5	200.00	30.23	7.87	−9.17	17.51	153.56	0.00	0.00	0.00	0.00	200.00	0.00	27.80	0.00	6.13	166.07
6	2	133.72	15.04	75.81	10.59	−25.12	57.40	0.00	131.80	0.00	1.93	0.00	0.00	92.32	0.00	42.17	−0.77
6	4	143.09	14.09	11.61	3.98	83.39	30.01	0.00	0.00	0.00	143.09	0.00	0.00	24.49	0.00	117.83	0.77

**Table 12** Charge (k€) of Generators and Loads on Garver's 6-Bus Test System for the Case of an Annualized Transmission Cost (TC) of 340 k€

Pricing method	Tracing method										Total charge (k€)
	GGDF method, 30% TC = 102.00 k€				GLDF method, 70% TC = 238.00 k€						
	Generators (k€)				Loads (k€)						
	G1	G3	G6	Sum	L1	L2	L3	L4	L5	Sum	
MW-m original	12.5434	35.7907	53.6659	102.0000	26.5042	55.3629	9.4309	64.5430	82.1591	238.0000	340.0000
Unused abs	17.1873	32.1239	52.6888	102.0000	27.5294	56.1915	8.1841	74.0657	72.0292	238.0000	340.0000
Unused ZCF	26.7338	38.1995	37.0667	102.0000	12.5993	65.4471	5.4240	97.3293	57.2004	238.0000	340.0000
Unused reverse	81.8498	114.9236	−94.7734	102.0000	−76.8049	−40.5952	−19.6660	480.1350	−105.0689	238.0000	340.0000
Used abs	17.2940	35.2364	47.7372	100.2677	28.2416	59.4505	9.0335	68.9989	77.3773	243.1018	343.3694
Used ZCF	16.0985	29.0443	30.1576	75.3003	11.0291	51.6081	4.4067	61.2720	51.9568	180.2727	255.5730
Used reverse	14.9029	22.8522	12.5779	50.3330	−6.1833	43.7656	−0.2201	53.5452	26.5363	117.4436	167.7766
Postage stamp	20.1316	44.7178	37.1506	102.0000	25.0526	75.1579	12.5263	50.1053	75.1579	238.0000	340.0000

**Figure 3** Generators charges on Garver's 6-bus system using (a) GGDF, (b) Bialek upstream, and (c) the minimum power distance method. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

Next, the student can press the button “Plot” so as to present in chart form the charge of network users as a percentage of the total cost (see Fig. 2). Finally, the student can press the button “Save results” so as to save the results of the TPS in a Microsoft Excel file.

## RESULTS AND DISCUSSION

The students are requested to use TPS in order to investigate the three tracing methods and the eight transmission pricing methodologies on two test systems: (1) Garver's 6-bus system and (2) IEEE RTS 24-bus system.

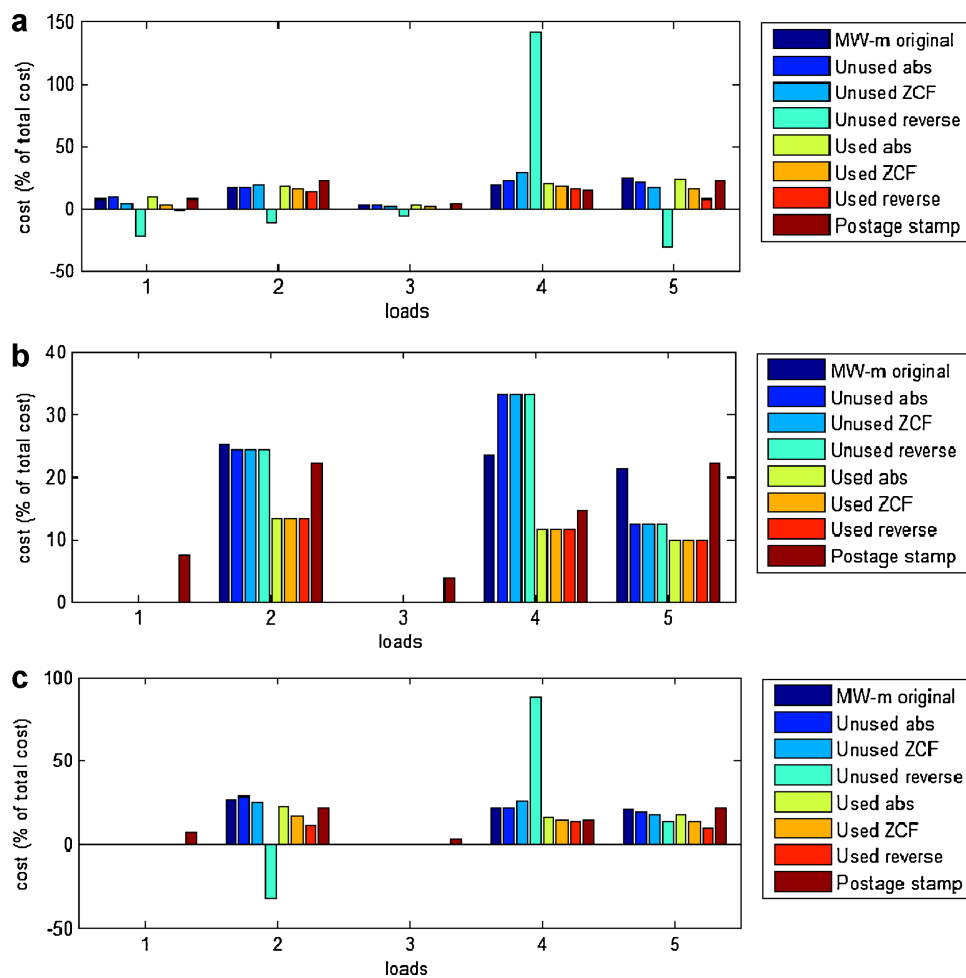
### Garver's 6-Bus System

Garver's test system is a power system with six buses, three generators, and six transmission lines [23]. The total peak load is 760 MW, while the total installed generation capacity is 1,110 MW. The data of an expanded form of this system can be found in Tables 1 and 2. This expanded system has been already studied in “Educational Example” Section using one

tracing and one pricing method. In this section, the same system will be studied considering eight transmission pricing and three tracing methodologies.

Table 10 shows the generators contribution to line flows using the following three methods: (1) GGDF, (2) Bialek upstream, and (3) minimum power distance method. For example, Table 10 shows that when using GGDF method the contribution of generators to the 16.60 MW flow in line 1–2 is as follows: 41.24 MW are coming from the generator of bus 1, 5.38 MW from generator of bus 3, and –30.02 MW from the generator of bus 6 (i.e., the generator of bus 6 creates counter flow of 30.02 MW).

Table 11 shows the loads contribution to line flows using the following three methods: (1) GLDF, (2) Bialek downstream, and (3) minimum power distance method. For example, Table 11 shows that when using Bialek downstream method the contribution of loads to the 16.60 MW flow in line 1–2 is as follows: 16.36 MW of the flow in line 1–2 are intended to serve the load of bus 2, 0.24 MW are intended to serve the load of bus 4, while 0.0 MW from the flow of line 1–2 are intended to serve the loads of buses 1, 3, and 5.

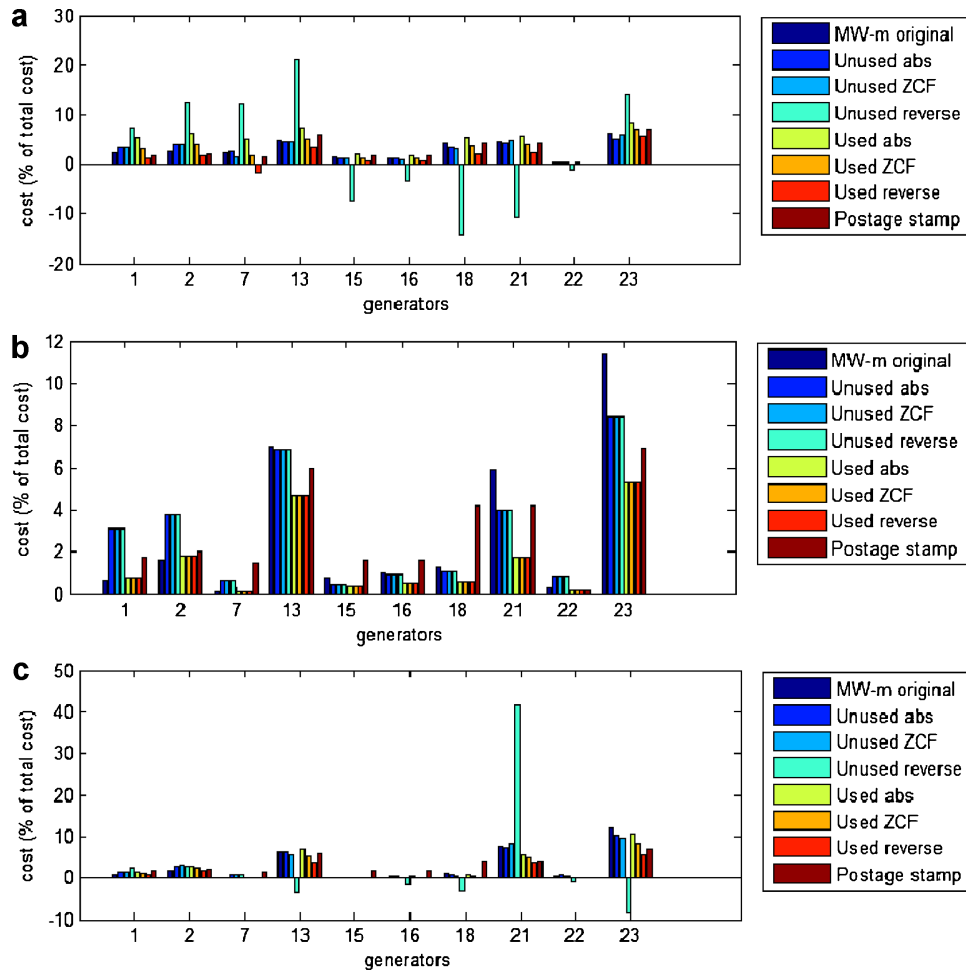


**Figure 4** Loads charges on Garver's 6-bus system using (a) GLDF, (b) Bialek downstream, and (c) the minimum power distance method. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

The annualized cost of investment for all the transmission lines is 340 k€, which is the total cost (TC) that has to be charged to the network users. Let us suppose that the student selects that 30% of TC is charged to generators and the rest 70% is charged to the loads, which means that 102 k€ is charged to generators and the rest 238 k€ is charged to loads. Table 12 shows the charge of generators using the GGDF tracing method and the charge of loads using the GLDF tracing method. The following conclusions are drawn from Table 12:

- (1) Five out of eight transmission pricing methods fully recover the total cost for generators (102 k€) using the GGDF tracing method. These pricing methods are: (a) MW-mile, (b) unused abs, (c) unused ZCF, (d) unused reverse, and (e) postage stamp method.
- (2) Three transmission pricing methods under-recover the total cost for generators (102 k€) using the GGDF tracing method. These pricing methods are: (a) used abs, (b) used ZCF, and (c) used reverse, among which the used reverse method recovers only 50.333 k€ out of 102 k€.
- (3) Five out of eight transmission pricing methods fully recover the total cost for loads (238 k€) using the GLDF tracing method. These pricing methods are: (a) MW-mile, (b) unused abs, (c) unused ZCF, (d) unused reverse, and (e) postage stamp method.
- (4) Two transmission pricing methods under-recover the total cost for loads (238 k€) using the GLDF tracing method. These pricing methods are: (a) used ZCF and (b) used reverse, between which the used reverse method recovers only 117.4436 k€ out of 238 k€.

The results of Table 12, as a percentage of the total cost, are shown in Figures 3a and 4a, which are also produced by TPS. For example, Table 12 shows that the generator of bus 1 is charged 12.5434 k€ using the GGDF tracing method and the original MW-mile pricing method, which corresponds to  $12.5434/340 = 3.69\%$  of the total cost of 340 k€, so in Figure 3a, this 3.69% value is depicted for the original MW-mile pricing method for the generator of bus 1.



**Figure 5** Generators charges on IEEE RTS 24-bus system using (a) GGDF, (b) Bialek upstream, and (c) the minimum power distance method. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]



### IEEE RTS 24-Bus System

The IEEE RTS [24] is a network with 24 buses, 38 lines and transformers, and 32 generating units. The total peak load is 2,850 MW and the total installed generation capacity is 3,405 MW.

The annualized cost of investment for all the transmission lines is 139,105 k€, which is the total cost (TC) that has to be charged to the network users. Let us suppose that the student selects that 30% of TC is charged to generators and the rest 70% is charged to the loads, which means that 41,731.5 k€ is charged to generators and the rest 97,373.5 k€ is charged to loads. Figure 5 presents the generators charges on IEEE RTS 24-bus system using (a) the GGDF, (b) the Bialek upstream, and (c) the minimum power distance method. Figure 6 shows the loads charges on IEEE RTS 24-bus system using (a) the GLDF, (b) the Bialek downstream, and (c) the minimum power distance method.

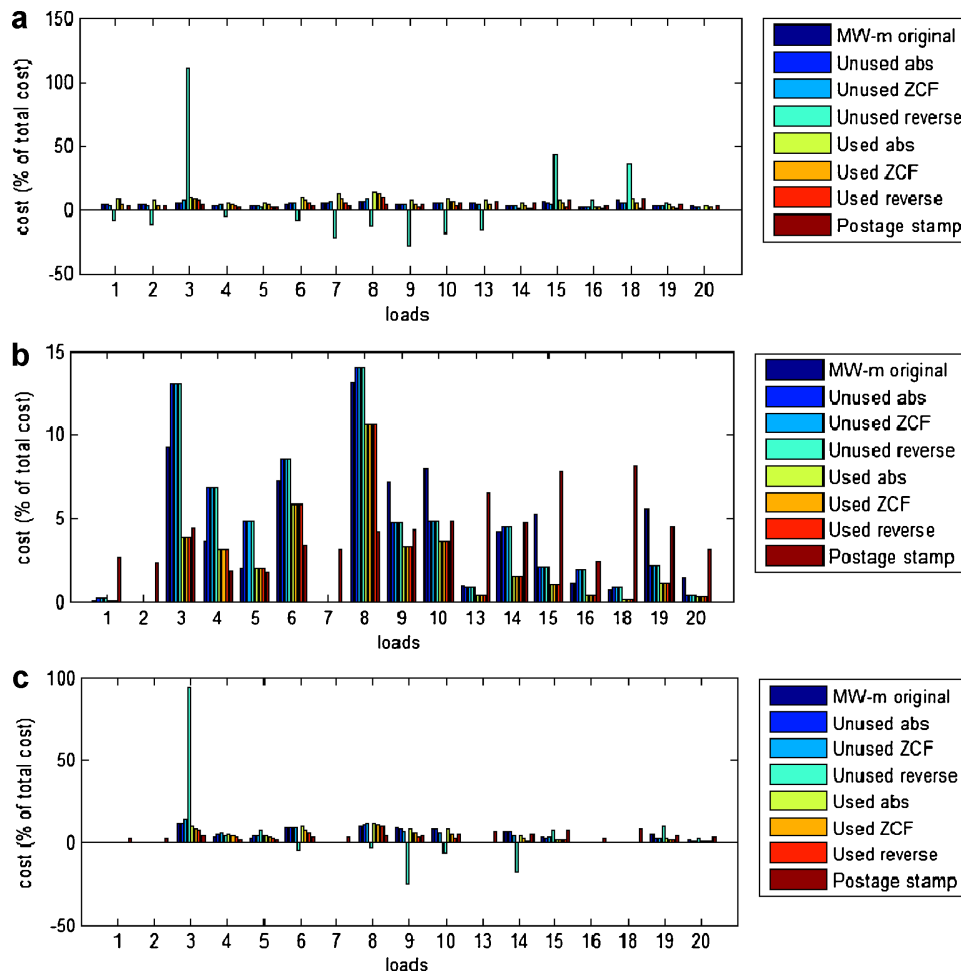
The software has also been tested with a power system consisting of 2,383 buses and the transmission pricing problem was successfully solved in less than 20 s CPU time, where

almost 50% of that time was spent for the solution of DC OPF problem. This performance is similar to the performance of MATPOWER software in the solution of DC OPF problem with MATLAB optimization toolbox [25].

### Students Feedback

The above transmission pricing software has been used during the last 3 years at the National Technical University of Athens to help teaching the undergraduate course of power system economics. The students learnt very easy to use this software thanks to its powerful user interface and due to the experience of students with computers and MATLAB.

This computer-aided education method was assessed both formally with student evaluations and informally from discussions with students. It should be noted that students rated the software and the education material positively and course evaluations were higher after these tools were introduced. In fact, this software helps students clarify the differences and the impact of the eight transmission pricing and the three tracing methodologies on transmission cost allocation.



**Figure 6** Loads charges on IEEE RTS 24-bus system using (a) GLDF, (b) Bialek downstream, and (c) the minimum power distance method. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

An increase was observed in the number of students who are selecting the power system economics course to make a Diploma dissertation with a focus on algorithm development using MATLAB. These students not only take care of the algorithm part of their dissertation but also the software and the GUI. All these findings provide incentives for implementing similar computer-aided education methodologies to other courses as well.

## CONCLUSIONS

This article presents a novel approach to education in the field of transmission pricing. A computer program was developed to present the effects of eight transmission pricing and three tracing methodologies on transmission cost allocation. The computer program was implemented in MATLAB, because it is a powerful programming environment that integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. The GUI makes the software very friendly to the students. Moreover, the students can not only see the final solution, that is, the cost allocation among the network users, but also important intermediate results, for example, the contribution of network users to transmission line flows. The use of the program is presented for two different power systems: Garver's 6-bus and IEEE RTS 24-bus system. These test examples help the students understand the impact on transmission cost allocation of various parameters, for example, the location of the user, the tracing method used, the pricing or not of the counter flows, and the generation bid. The visualization of the parameters and the results of this transmission pricing software enable an easier and deeper insight into the impact of transmission system parameters and pricing methodologies on transmission cost allocation.

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